



Reg. No. :

Name :

**Fifth Semester B.Tech. Degree Examination, November 2014
(2008 Scheme)**

08.502 : ADVANCED MATHEMATICS AND QUEUING MODELS (RF)

Time : 3 Hours

Max. Marks : 100

Instruction : Answer **all** questions of Part **A** and **one** full question **each** from Module I, Module II and Module III of Part **B**.

PART – A

- Find all basic solutions of the following system $2x_1 + x_2 + 4x_3 = 11$ and $3x_1 + x_2 + 5x_3 = 14$
- Define slack and surplus variables. Give example.
- What are the basic steps in PERT/CPM techniques ?
- Tasks A, B, C, ..., H, I constitute a project. The notation $X < Y$ means that the task X must be finished before Y can begin with this notation, construct the network diagram having the following constraints, $A < D, A < E, B < F, D < F, C < G, C < H, F < I, G < I$.
- Find the LU decomposition of the matrix.

$$A = \begin{bmatrix} 1 & -1 & -1 \\ 3 & -4 & -2 \\ 2 & -3 & -2 \end{bmatrix}$$



- Let $H = \{ (a - 3b, b - a, a) / a, b \in \mathbb{R} \}$. Show that H is a subspace of \mathbb{R}^3
- Find a unit vector orthogonal to $(1, 1, 1)$ and $(1, 2, -3)$ in \mathbb{R}^3 .
- What is queue discipline and give three different possible queue disciplines ?
- If λ and μ are, respectively, the arrival and service rates of an $(M | M | 1) : (\infty | FIFO)$ queueing model, show that the average number of customers in the system $L_s = \frac{\rho}{1 - \rho}$ where $\rho = \frac{\lambda}{\mu}$.
- In an $(M | M | C) : (\infty | FIFO)$ queueing model if $\lambda = 15/\text{hour}$, $\mu = 6/\text{hour}$ and $c = 3$ find P_0 . **(10×4=40 Marks)**



PART - B

Module - I

Answer **one** full question.

11. a) Use simplex method to solve the following LPP.

$$\text{Maximize } z = x_1 - x_2 + 3x_3$$

Subject to the constraints

$$x_1 + x_2 + x_3 \leq 10$$

$$2x_1 + 0x_2 - x_3 \leq 2$$

$$2x_1 - 2x_2 + 3x_3 \leq 0 \text{ and}$$

$$x_1, x_2, x_3 \geq 0.$$

10

- b) The utility data for a network is given below. Determine the total float, free float and independent float of the network.

10

Activity	0-1	1-2	1-3	2-4	2-5	3-4	3-6	4-7	5-7	6-7
Duration	2	8	10	6	3	3	7	5	2	8

12. a) Solve by simplex method

$$\text{Maximize } z = 30x_1 + 40x_2$$

$$\text{Subject to } 60x_1 + 120x_2 \leq 12,000$$

$$8x_1 + 5x_2 \leq 600$$

$$3x_1 + 4x_2 \leq 500$$

$$x_1, x_2 \geq 0$$

10



b) The following table gives the data for the activities of a small project

Job :	1-2	1-3	2-4	2-6	3-4	3-5	4-5	5-6
to :	1	5	3	1	8	2	5	2
tm :	4	10	3	4	15	4	5	5
tp :	7	17	3	7	26	8	5	8



i) Draw the network and find the expected project completion time.

ii) What is the probability that it would take 5 days more than the expected duration ?

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Module – II

13. a) Solve the equation $AX = B$ if $A = LU = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 1 \end{bmatrix} \begin{bmatrix} 3 & -7 & -2 \\ 0 & -2 & -1 \\ 0 & 0 & -1 \end{bmatrix}$ and

$$B^T = (-7, 5, 2)$$

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b) Find the spanning set for the null space of the matrix

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

10

14. a) Find the rank and basis of the row-space of the matrix $\begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{bmatrix}$ 7

b) Find the projection of $u = (1, -2, 3, -4)$ along $v = (1, 2, 1, 2)$ in \mathbb{R}^4 . 7

c) Show that the quadratic form $x^2 + 2y^2 + 3z^2 + 2xy + 2yz - 2xz$ is indefinite. 6



Module – III

15. a) Customers arrive at a one man barber shop according to poisson process with a mean arrival time of 20 minutes. Customer spend an average of 15 minutes in the barber's chair.
- What is the expected number of customers in the barber shop in the queue ?
 - How much time can a customer expected to spend in the barber shop ?
 - What is probability that the waiting time in the system is more than 10 minutes ?
 - What is the probability that there are more than 3 customers in the system ? 10
- b) A bank with a single server, there are 2 chairs for waiting customers. On an average one customer arrives every 10 minutes and each customer takes 5 minutes for getting served. Making suitable assumptions, find
- Probability that an arrival will get a chair to sit on
 - The probability that an arrival will have to stand
 - Expected waiting time of a customer. 10
16. a) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour.
- What is the probability that an arriving patient will not wait ?
 - What is the expected waiting time until a patient is discharged from the clinic ? 10
- b) A telephone exchange has 2 long distance operators. The telephone company finds that during the peak load, long distance call arrive in a poisson fashion at an average rate of 15 per hour. The length of service on these calls is approximately exponential with mean length of 5 minutes.
- What is the probability that a customer will have to wait for his long distance call during the peak hours of the day ?
 - If the customer will wait and are serviced in turn, what is the expected waiting time ? 10